Prediction of Low Speed Stability by Process Damping Theory

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1. Introduction

It is well known among machinists that taking low enough cutting speed increases dynamic stability so that chatter is prevented.

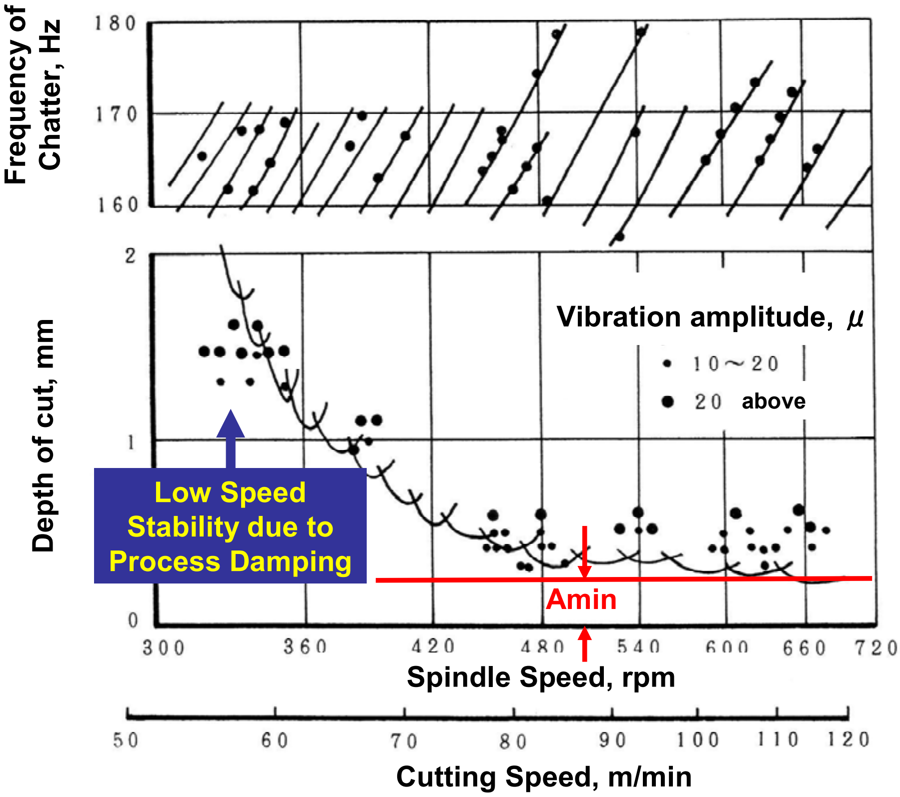

Fig. 1 Experimental result of stability border diagram measured in turning test. (Lines represent results of computational simulation)

Principal direction of natural mode of vibration of tool supporting structure: depth of cut direction, natural frequency: 150 Hz.

Feed rate: 0.115 mm/rev, flank wear width: Vb = 0 – 50 μm, work piece material S35C plain carbon steel.

[T. Takemura, Research on Avoidance of Chatter in Turning, Doctor Dissertation submitted to Kyoto University, 1977 1)]

When two parameters, cutting speed and depth of cut are mapped in a Cartesian coordinate as illustrated in Fig. 1 and amplitudes of vibration are marked as measured in cutting at selected combinations of two parameters, there can be defined unconditional stability limit Amin of the depth of cut below which system is stable so that chatter does not occur irrespective of cutting speed.

It is noticed that as cutting speed is selected lower, greater depth of cut is possible without onset of chatter. This phenomenon, called Low Speed Stability, is understood to be caused by "Process Damping" which induces increased amount of stabilizing force on cutting tool at lower cutting speed.

In academic circle, historical study of Das and Tobias 2-4) concluded that the effect is due to instant oscillation of the orientation of total cutting force acting on the cutting edge according to the slope of the inner modulation on which vibrating cutting tool traces.

The concept described in the hypothesis of Das and Tobias has been later confirmed experimentally, based on which present study tries to formulate a method of predicting the upward expansion of stability limit at lower cutting speed range.

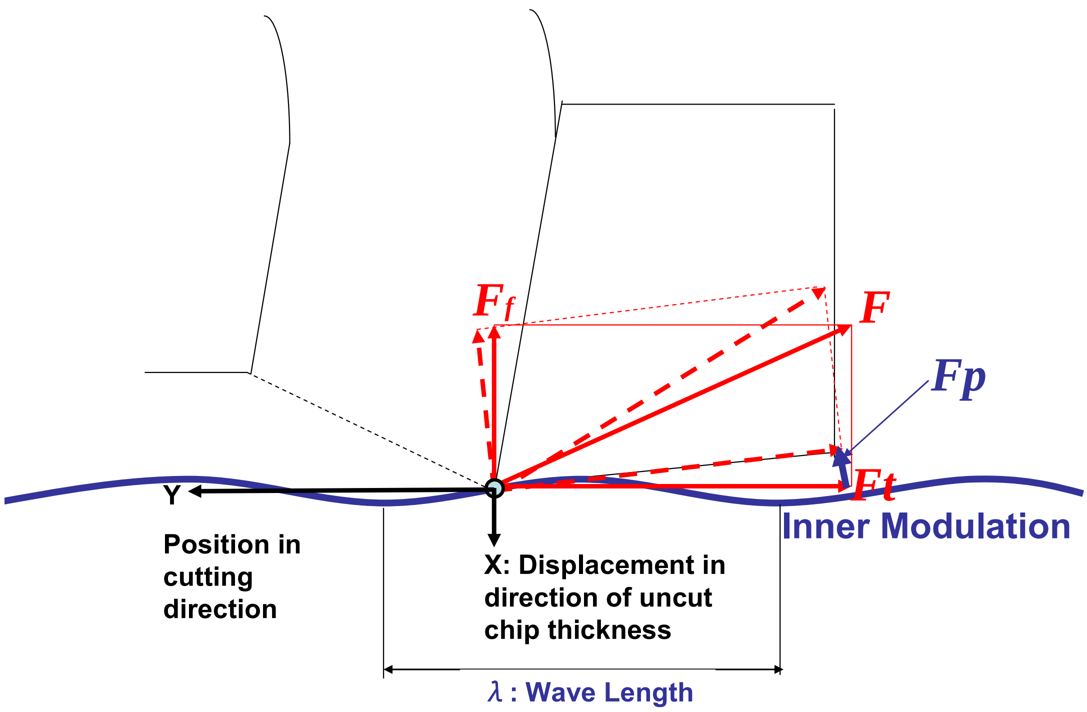

Fig. 2 Hypothesis of Process Damping after Das and Tobias.

2. Mathematical Model of Process Damping.

2.1 Hypothesis by Das and Tobias

Their conclusion was that the damping force that suppresses chatter is generated by instantaneous oscillation of the orientation of total cutting force according to the slope of inner modulation.

As illustrated in Fig. 2, hypothesis noted in the above, sometimes referred to as "Imaginary Part Effect of Inner Modulation" can be modeled as follows:

Generation of $\boldsymbol{F_p}$ is in proportion to the width of cut b, modeled by:

$$F_p = bF_t \frac{\left(\frac{dX}{dt}\right)}{\left(\frac{dY}{dt}\right)}$$

$$= \frac{bF_t(-j2\pi(Frequency\_of\_Vibration\,in\,Hz)X)}{(CuttingSpeed\_in\_mm/\sec)}$$

… (1)

Since wavelength $\boldsymbol{\lambda}$ is:

$$\lambda = \frac{(Cutting\ Speed\ in\ mm/\sec)}{(Frequency\ of\ Vibration\ in\ Hz)} \qquad \text{… (2)}$$

$\boldsymbol{F_p}$ in the above equation (1) is reduced to:

$$F_p = \frac{bF_t(-j\,2\,\pi)\,X}{\lambda} \qquad \text{… (3)}$$

The Stiffness Frequency Response Function $\boldsymbol{T_{px}}$ is then,

$$T_{px} = \frac{bF_p}{X} = \frac{-j2\pi bF_t}{\lambda} \qquad \textbf{... (4)}$$

Cutting force components are obtained by generic two-dimensional cutting tests and related to uncut chip thickness *h* and width of cut *b* as illustrated below.

2.2 Experimental Verification of Process Damping Model

$\boldsymbol{F_t}$ in equation (4) is modeled as

$$F_t = K_{oy}\, h$$

, $\boldsymbol{K_{oy}}$: Static tangential force coefficient

$\boldsymbol{T_{px}}$ is then:

$$T_{px} = \frac{-j2\pi K_{oy}\, b\, h}{\lambda} \qquad \textbf{... (7)}$$

Experimental proof for the generation model of process damping as described in the above equation (7) is presented in a result of measurement, Fig.3 conducted in turning tests using tool support system having variable natural frequencies (146 to 1167Hz) at variable feed rates ***f*** (0.05 to 0.2 mm/rev) and depths of cut ***d*** (0.1 to 0.6mm).

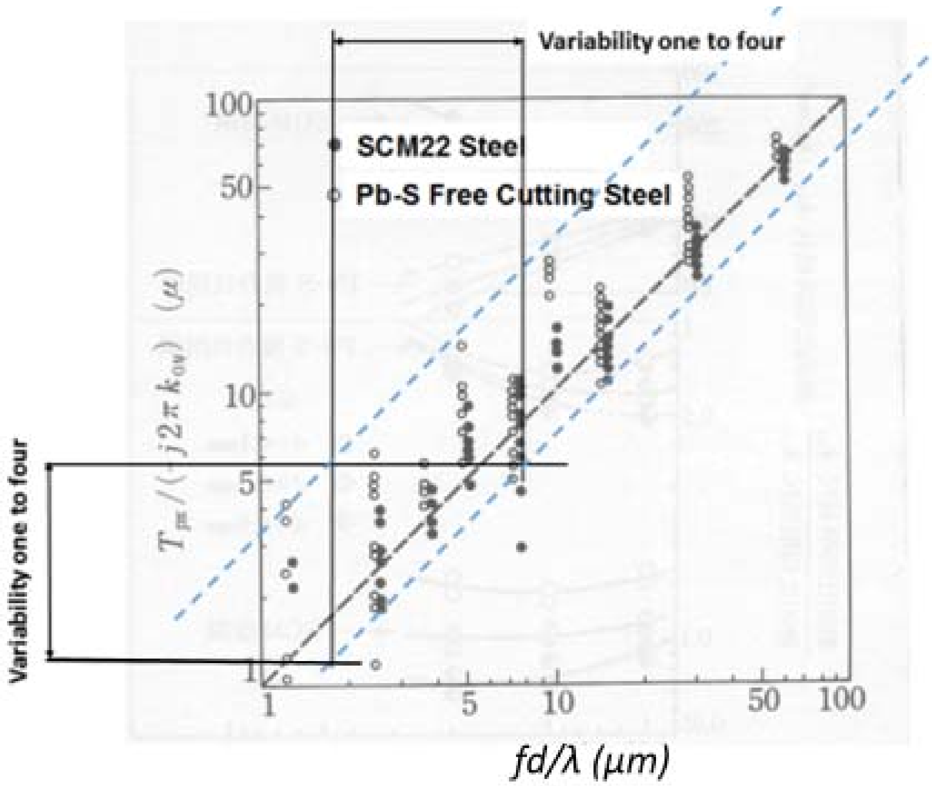

Fig.3 Experimental value of imaginary part effect of inner modulation measured in turning test. *Koy=Ft/bh, f=h, d=b*
[Hoshi 5), 6)]

3. Coordinating Process Damping in Stability Border

3.1 Stability Border without Process Damping

Referring to conventional stability border model illustrated in Fig. 4, the so called "Maximum Real Negative Part" of structure dynamics Gmin defines "Unconditional Stability Limit" Amin by following equation:

$$A_{min} = \frac{1}{2K_{fc}G_{min}} \qquad \text{… (8)}$$

Cutting dynamics is represented in the figure by a vertical line marked red.

Equation (8) is valid only when overlap factor is unity 1 (Machining situations such as in parting or grooving of width b, or end milling of axial depth of cut b). In Machining situations where the overlap factor is less than 1, the cutting dynamics line (vertical line colored in red in next figure 4) is reduced to a curve inscribed on the left-hand side of the vertical red line, the stability border is reached by taking width of cut b greater than A_{min} calculated by equation (8).

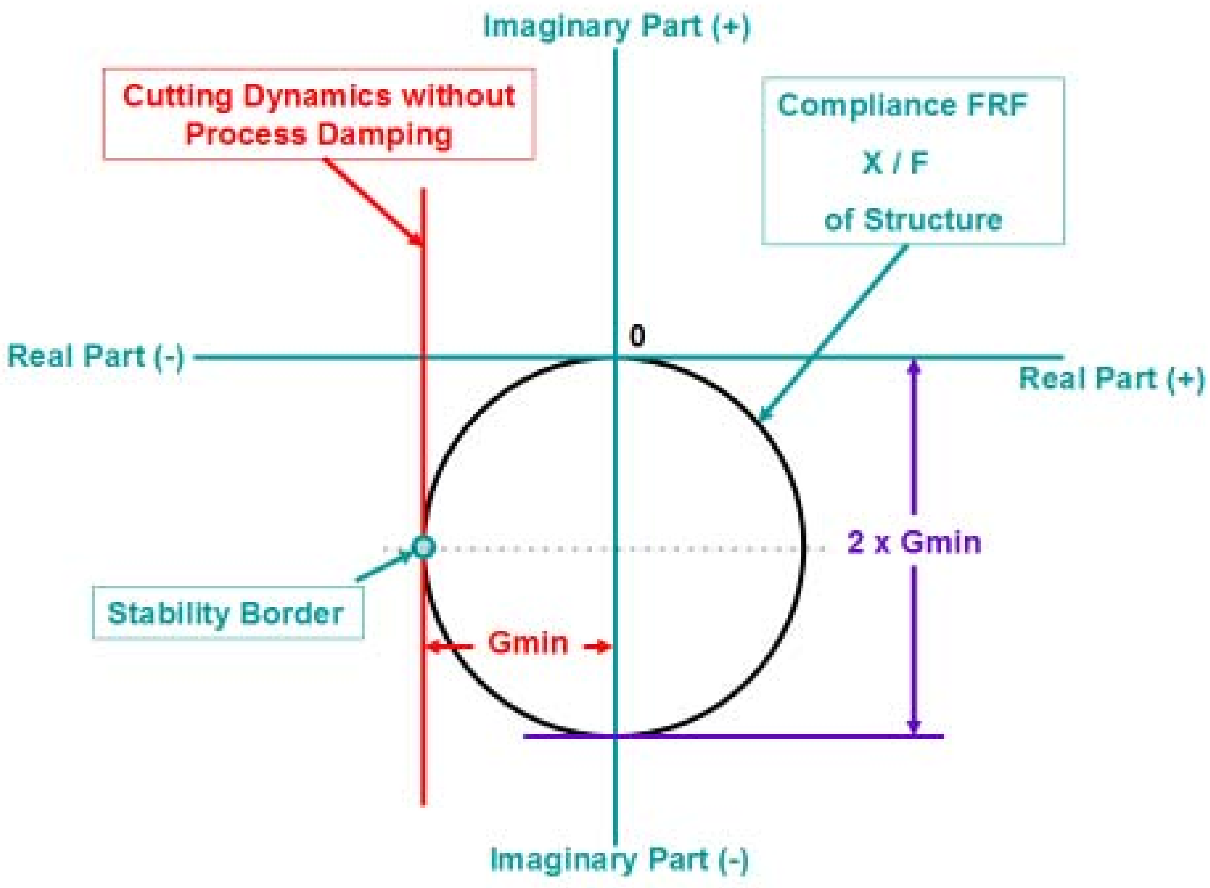

Fig. 4 Conventional stability border model in compliance vector diagram.

3.2 Stability Border considering Process Damping

In a stiffness vector diagram Fig.5, the cutting dynamics without considering process damping is represented by a smaller red circle at right upper corner.

Stiffness FRF F/X of machine structure is represented by a horizontal straight line tangential to the small circle. It used to be represented by a circle in the compliance vector diagram in Fig.4, but in the stiffness vector diagram Fig.5 represented by a straight line. The distance of the straight

line from the origin O of the coordinate is the minimum stiffness of the structure and it is the inverse of the maximum compliance 2Gmin shown in Fig.4.
Center of the red circle represents inner modulation, while the circle itself corresponds to the outer modulation having variable phase angle to the inner modulation.
When considering Process Damping, the circle center is shifted downward by the amount of Tpx, therefore stability border is reached only by taking greater width of cut b which is the Stability Limit with Process Damping Alim.

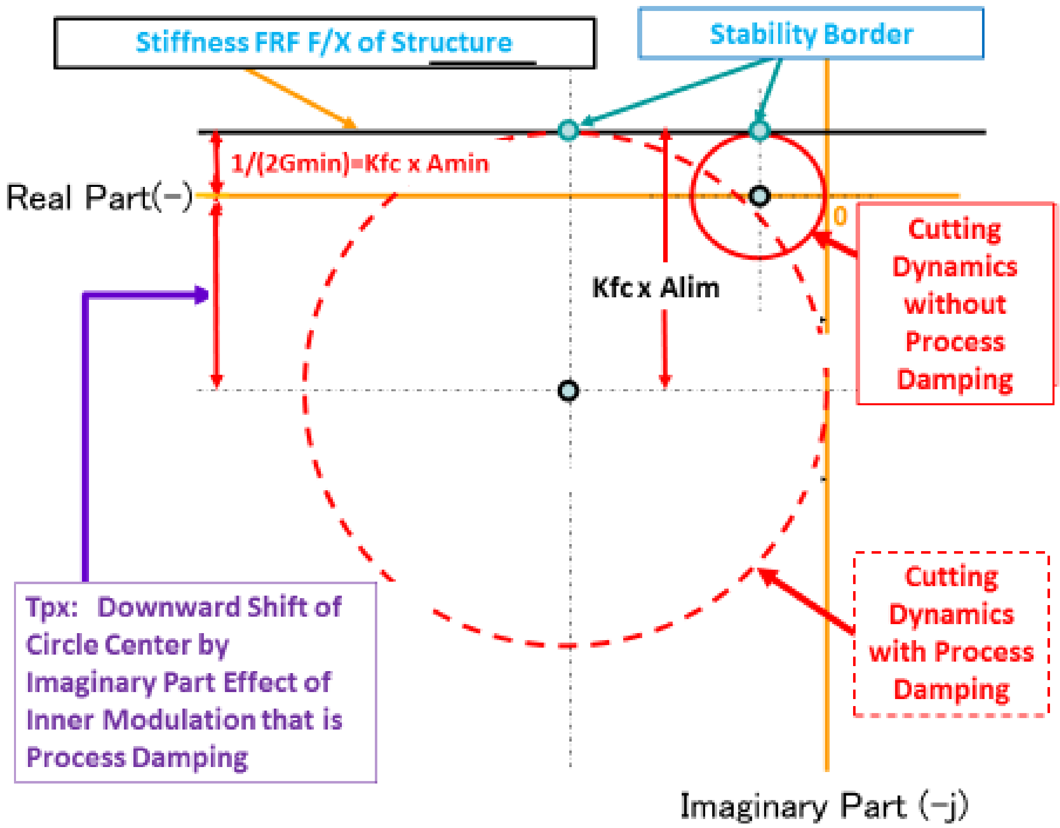

Fig. 5 Stability border model in stiffness vector diagram.

The distance between the down shifted circle center and the horizontal straight line at the top representing the structure stiffness is the width of cut ***b= Alim*** at the stability border multiplied by ***Kf***c.

$$F_t = K_{tc} \times b \times h \quad \ldots (10)$$

Using equation (10) and replacing b by Alim, T_{px} in equation **(4)** is

$$T_{px} = \frac{(-j)2\pi K_{tc} A_{lim} h}{\lambda} \qquad \text{… (11)}$$

Noting in the stiffness diagram Fig.6, an equality KfcAlim= KfcAmin+Tpx is holding and replacing ***b*** with $\boldsymbol{A_{lim}}$, and dividing both sides of the equation by Kfc and introducing a variable γ that stands for tangential to feed dynamic cutting force ratio,

$$\gamma = \frac{K_{tc}}{K_{fc}}$$

condition for the outer modulation reaching stability border is represented by:

$$A_{lim} + \frac{T_{px}}{K_{fc}} = A_{min} + \frac{2\pi\gamma A_{lim} C_s h}{\lambda} \qquad \text{… (12)}$$

$$A_{lim} = \frac{A_{min}}{\left(1 \quad \frac{2\pi\gamma C_s h}{\lambda}\right)}$$

Since wavelength λ is

$$\lambda = \frac{\pi \times (Diameter) \times (rpm)}{60\,(Frequency)}$$

Finally, stability border including process damping:

$$A_{lim} = \frac{A_{min}}{\left(1 - \frac{120 \times (Natural\ Frequency, Hz)\gamma \times C_s h}{(Tool\ Diameter, mm) \times (Spindle\ Speed, rpm)}\right)}$$

… (13)

There occurs an asymptotic spindle speed $\boldsymbol{S_{as}}$ to which $\boldsymbol{A_{lim}}$ approaches infinity when denominator of (13) reduces to zero, namely:

$$S_{cs} = 120(Natural\ Frequency, Hz)\gamma C_s h / (Tool\ Diameter, mm)$$

… (14)

The parameter Cs in the above needs to be 1 when applied to turning and boring operations where uncut chip thickness ***h*** is equal to feed rate ***f***, and number of tool engaged is always one.

For milling application; the equation needs to be adjusted as noted below. Referring to Fig.6, ratio of average uncut chip thickness to h is represented by a new parameter **Cs:**

$$C_s = \frac{\int_{EA}^{DA} (\cos\theta)\, d\theta}{\int_{EA}^{DA} d\theta} = \frac{180(\sin DA - \sin EA)}{(DA - EA)\pi} \qquad \textbf{… (15)}$$

Feed rate h (mm/tooth) in equation (14) has to be replaced with average uncut chip thickness h x **Cs**.

When disengage angle DE is fixed at 90deg, and calculating **Cs** for engage angle EA=-90 to +90deg, Cs is found as illustrated in Fig.8 to be maximum 0.742 at engage angle EA=40 deg .

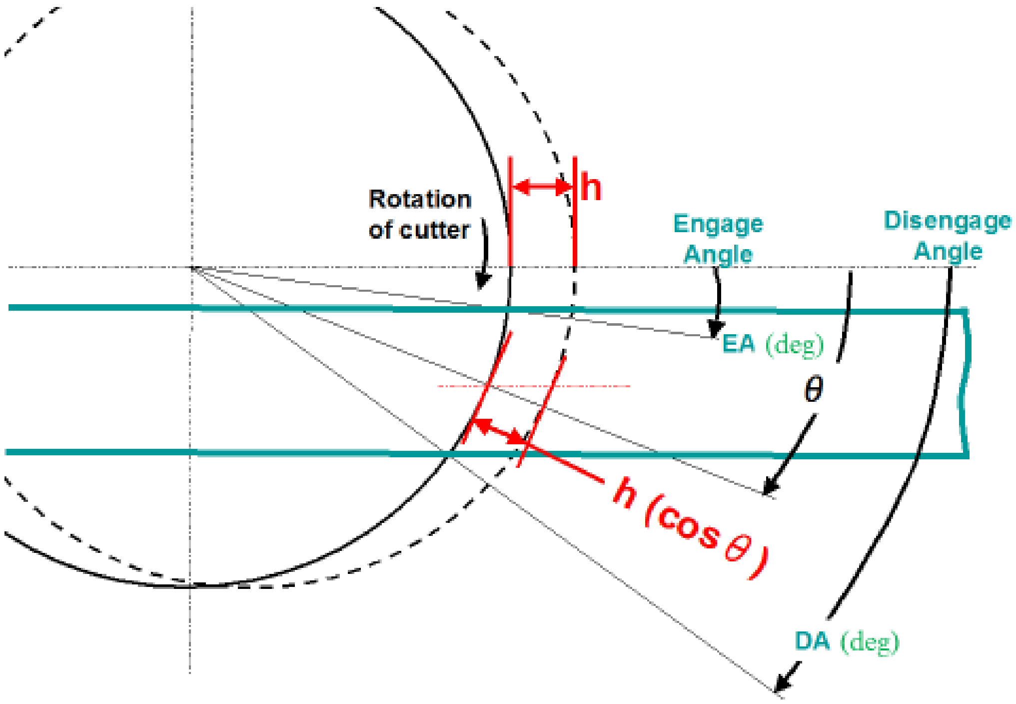

Fig. 6 Average values for milling.

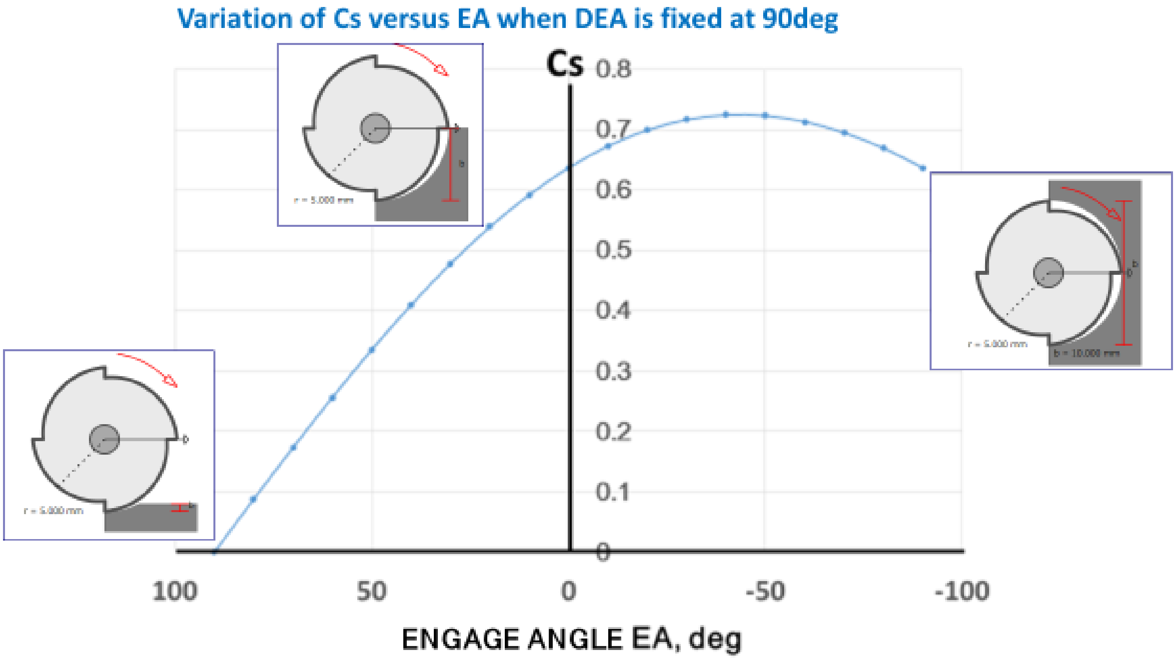

Fig.7 Variation of **Cs** for engage angle EA=-90 to +90deg
when disengage angle DA is fixed at 90deg

In rough milling applications where engage angle EA tends to be large negative value (right side of Fig.7), Cs values tend to be high 0.63-0.72 that may cause Sas to be high. Process damping by this Das, Tobias (1960-64) model seems to be effective in rough machining situations.
On the other hand, for finishing applications, EA is going to be a small angle (left side of the figure) Cs and hence Sas assumes very small value making the effect expected to be negligible small.

As cutting speed gradually reduces and approaches the recommended cutting speed defined by equation (16), unconditional stability border A_{lim} increases as illustrated in Fig.8.

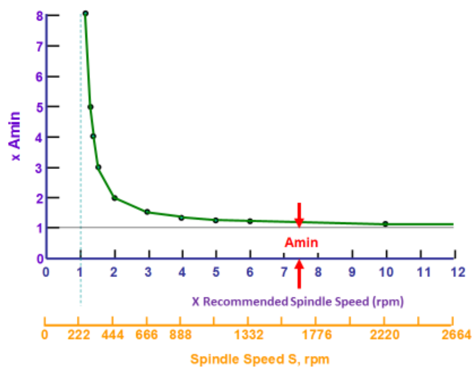

Fig. 8 Profile of low speed stability.

4. Numerical Example

Work material: AL6061-T6
Tool: Diameter 20,
$\gamma = K_{tc} / K_{fc}$ = 1574.7/698.8 = 2.2563

Cutting conditions:
$f = h$ = 0.05mm/tooth
Frequency: 1,000Hz,
Engage Angle (EA)=30deg, Disengage Angle (DA)=90deg
Cs=180(1-Sin30deg)/(90-30) π=0.478

$$S_{as} = 120(Natural\ Frequency, Hz)\gamma C_s h/(Tool\ Diameter, mm)$$
= 120 x 1000 x 2.2563 x 0.478x0.05/20
= 323rpm

5. Concluding Remarks

Referring to hypothesis of process damping mechanism proposed by Das and Tobias during 1964 to 1967 and later experimentally confirmed in 1972, a method has been investigated for predicting increased stability at lower cutting speed that prevents onset of regenerative chatter.

According to the mathematical model constructed for the hypothesis, effect of process damping increases in proportion to the magnitude of total cutting force in tangential direction, frequency of vibration and in inverse proportion to cutting speed.

As cutting speed is reduced, effect of process damping increases up to a point beyond which chatter can no longer occur at any large widths of cut. This lower limit of speed is represented by asymptotic spindle speed S_{as} whose value can be calculated using equation (14) prepared by the study.

Also the profile of stability border can be calculated as shown in Fig.8 by which the unconditional stability limit A_{min} defined by regenerative chatter theory is shifted upward in lower speed range.

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End of text

$$T_{px} = \frac{(-j)2\pi K_{tc} A_{lim}(h_0 + h)}{\lambda}$$